Dynamic Interaction of Floor - Light Steel - Equipment System - A Sensitivity Approach

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ABSTRACT

A sensitivity approach to study the dynamic interaction of floor-light steel-equipment system is presented. The analysis is conducted in two phases: 1) free-vibration sensitivity analysis, and 2) seismic response sensitivity analysis. Particular attention is given to the effects of the light-steel that connects the equipment to the floor. The damped frequencies of the structure (floors) equipment system are obtained using the complex interpotation on a unit circle together with the powerful Jenkins-Traub algorithm. The sensitivity of the frequencies to parameter variations is investigated. The response of the system to strong ground motion is studied. The sensitivity equations for the seismic response are developed and solved simultaneously with the system equations of motion to obtain the response history sensitivity function. The results obtained herein are used to examine the conservatism of the decoupling criteria of the CSA N289.3 standard. Finally, conclusions and recommendations are given.

1.0 INTRODUCTION

In seismic analyses of nuclear power plant structures, because typical structural systems are, in many cases, dynamically coupled, the required size of an integrated seismic model may become so large that it is impractical. To keep the size of the model within practical limits, the physical interconnection between structural system and equipment or mechanical system is often disregarded. Significant differences may, however, occur in the estimation of the dynamic behaviour of the uncoupled system from the dynamic behaviour of the integrated system (coupled). Such differences, herein termed dynamic interaction effects, may cause the response of the coupled system to be significantly different from the uncoupled dynamic responses of the structural system and the component (equipment) subsystem. Dynamic interaction effects are particularly important in nuclear equipment design because of safety requirements and, furthermore, because components are attached to rather elaborate support systems and other components, either directly such as pump motor drive assemblies or indirectly through complex piping systems such as the primary coolant loops[1]. A single seismic model of such large systems may become unmanageable, especially at the preliminary design stage. Disregarding interaction effects at the preliminary design stage, although expedient, may not ensure reasonable conservatism. On the other hand, a detailed seismic model may be neither feasible nor desirable. An alternative approach is to include a simplified model of the subsystem, which correctly represents its stiffness and mass effects, in the primary system dynamic model. The resulting motion of the subsystem support locations are then used as forcing functions for separate and more detailed subsystem analysis. This approach enables the analysts to use more detailed subsystem mathematical models when performing either spectrum analysis or time history analysis of the subsystem, while accounting for dynamic interaction between the system and the subsystem. Methods for decoupling the responses of light subsystem from massive structural system have been suggested by Pal et-al[1], Gerdes[2], and Aziz et-al[3]. The decoupling criterion adopted by the CSA standard CAN3-N289.3-M81 [4] is that suggested in Ref 3. In this criteria, if the subsystem mass is larger than 1 percent of the structural mass, decoupling is acceptable if

$$\mu \leq \frac{1.1\,\theta - 1}{10\,\theta} \quad , \text{ for } \quad \theta \geq 1.0$$

or

$$\mu \leq \frac{1.1 - \theta}{10\,\theta} \quad , \text{ for } \quad \theta \leq 1.0$$

are satisfied. Where μ and θ are the ratios of the modal mass and of the modal frequency squared respectively. It is the objective of this paper to present a novel approach in studying the problem of dynamic interaction of structure-equipment system, to examine the validity of code formulae for decoupling, and to investigate the effects of light-steel on the dynamic interaction of the structure-equipment system.

2.0 MATHEMATICAL MODEL AND EQUATIONS OF MOTION

The simplified lumped mass model for the structural-equipment system is shown in Fig (1)a. The equipment model is shown in Fig (1)b and the equipment-light steel model in Fig (1)c.

The equations of motion of the system shown in Fig (1)a can be written as follows:

$$[M]\ \ddot{\underset{\sim}{Y}} + [C]\ \dot{\underset{\sim}{Y}}\ [K]\ \underset{\sim}{Y} = -[M]\ \underset{\sim}{D}\ ag \tag{1}$$

In which [M],[C], and [K] are mass, damping, and stiffness matrices respectively. $\underset{\sim}{D}$ is a vector with elements =1 and ag is the ground acceleration.

2.1 Free-Vibration and Sensitivity Formulation

The equations of free-vibrations can be obtained by replacing the right hand side term of Eq. (8) by $\underset{\sim}{0}$ (zero vector), i.e.:

$$[M]\,\underset{\sim}{\ddot{Y}} + [C]\underset{\sim}{\dot{Y}} + [K]\,\underset{\sim}{Y} = \underset{\sim}{0} \qquad (2)$$

Eq. (2) is transformed into a generalized eigenvalue problem using Laplace-Carson integral transformation, i.e. multiply the equation by $e^{-\lambda t}$, integrate each of its terms with respect to t between 0 and ∞, and then multiply it by λ (where λ is a variable in the complex plane). This leads to

$$(\lambda^2[M] + \lambda[C] + [K])\underset{\sim}{\overset{*}{Y}} = \underset{\sim}{0} \qquad (3)$$

where $\underset{\sim}{\overset{*}{Y}}$ is the vector $\underset{\sim}{Y}$ in the frequency domain. A non-trivial solution of Eq. (3) requires

$$\det\cdot(\lambda^2[M] + \lambda[C] + [K]) = 0 \qquad (4)$$

or

$$\det\cdot A(\lambda,\underset{\sim}{h}) = 0 \qquad (5)$$

where A is a matrix ploynomial in λ, λ's are variables in the complex plane, and $\underset{\sim}{h}$ is vector of parameters. The roots of Eq. (5) (eigenvalues) are obtained as follows:

1. The coefficients of the characteristic polynomial Eq. (5) are obtained by complex interpolation. Suppose n is the maximum degree of the polynomial (Eq. 5); then select (n+1) points uniformly spaced on a unit circle in the complex plane. The equivalent interpolation problem is subsequently transformed into a discrete Fourier transform [5]. It is of interest to note that the unit circle interpolation is numerically very stable. The computation time and accuracy of interpolation using various point sets were investigated in Ref. 5, and it was found that the unit circle interpolation was by some orders of magnitude superior both in accuracy and time requirements to any other method considered.

2. Once the characteristic polynomial has been obtained, any of the well known root finding algorithms could be used to generate the eigenvalues. The methods of Muller, Biarstow, Laguerre, and Jenkins-Traub were investigated in Ref. 6. It has been found that the Jenkins-Traub algorithm is the best, not only because it is reliable and fast but also because: a) it is globally convergent for real and complex roots, and b) the roots are found in increasing order of a magnitude so that the computation can be terminated when the dominant roots have been evaluated.

Sensitivity

Referring to Eq. (5), the sensitivity of the eigenvalue, λ_k, to some parameter q_i in the set h is defined as $\partial \lambda_k / \partial q_i$ and can be determined as follows:

Corresponding to the eigenvalue, λ_k, there exist right and left eigenvectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ such that

$$[A] \underset{\sim}{u} = \underset{\sim}{0} \tag{6}$$

and,

$$[A]^T \underset{\sim}{v} = \underset{\sim}{0} \tag{7}$$

Differentiating Eq. (6) with respect to q_i leads to

$$\frac{\partial [A]}{\partial q_i} \underset{\sim}{u} + \frac{\partial \lambda_k}{\partial q_i} \frac{\partial [A]}{\partial \lambda_k} \underset{\sim}{u} + [A] \frac{\partial \underset{\sim}{u}}{\partial q_i} = \underset{\sim}{0} \tag{8}$$

Premultiply Eq. (8) by $\underset{\sim}{v}^T$ and use Eq. (7) to get the sensitivity equation.

$$\frac{\partial \lambda_k}{\partial q_i} = - \underset{\sim}{v}^T \frac{\partial [A]}{\partial q_i} \underset{\sim}{u} / \underset{\sim}{v}^T \frac{\partial [A]}{\partial \lambda_k} \underset{\sim}{u} \tag{9}$$

The right, $\underset{\sim}{u}$, and left, $\underset{\sim}{v}$, eigenvectors can be obtained as follows:

1. Factor A as L U were L is the lower triangular and U is unit upper triangular. As λ is an eigenvalue then $\ell_{nn} = 0$, n being the dimension of A.

2. Normalizing $\underset{\sim}{u}$ and $\underset{\sim}{v}$ such that their last component is one. The u and $\underset{\sim}{v}$ are the solutions of

$$U \underset{\sim}{u} = \underset{\sim}{e}_n \tag{10}$$

and,

$$L_1 \underset{\sim}{v} = \underset{\sim}{e}_n \tag{11}$$

where $\underset{\sim}{e}_n$ is the unit vector with one in the location n and L_1 is the same as L with l_{nn} replaced by unity. Eqs. (10) and (11) can be solved for $\underset{\sim}{u}$ and $\underset{\sim}{v}$ by one back substitution each. Further, these vectors are generated only once and used in Eq. (9) for different parameters q_i in the set $\underset{\sim}{h}$. Now, let Δq_i, be a deviation in the nominal value of q_i ; then using the sensitivity function Eq. (9), the error or the corresponding deviation in the eigenvalue λ_k is given by

$$\Delta \lambda_k = \frac{\partial \lambda_k}{\partial q_i} \Delta q_i \tag{12}$$

2.2 Seismic Response and Sensitivity Formulations

An artificial earthquake record ag, may be used as input signals for the seismic analysis of structural-equipment system, Eq. (1). This equation could be solved directly by numerical integration of the coupled equations; however, in analyzing the earthquake response of linear structures, it is generally much more efficient to transform to a system of modal coordinates because the support motions tend to excite strongly only the lowest modes of vibrations. Thus good approximation of the earthquake response of systems having large number of degrees of freedom can often be obtained by carrying out the analysis for only a few modal coordinates. In this analysis the damping matrix is assumed to be proportional to the mass matrix to satisfy the orthogonality conditions.

Subroutine "DVERK" (based on Runge-Kutta methods) from IMSL is used to integrate the resulting uncoupled equations of motion in the time domain. The Runge-Kutta methods handles first order differential equations which means that the higher order equations must be reduced to first order before proceeding with the solution. Accordingly, the equations of motion can be reduced to the following form:

$$\dot{\underset{\sim}{z}} = \underset{\sim}{f}\,(\underset{\sim}{z}, t, \underset{\sim}{h}),\quad \underset{\sim}{z}(o) = \underset{\sim}{z}_o \tag{13}$$

where $\underset{\sim}{h}$ is the vector of parameters.

Sensitivity

To determine the sensitivity of $\underset{\sim}{z}$ to perturbations in q (where q is a parameter in $\underset{\sim}{h}$), Eq. (13) can be differentiated with respect to q to yield

$$\frac{\partial}{\partial q}\,\frac{\partial \underset{\sim}{z}}{\partial t} = \frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{z}}\,\frac{\partial \underset{\sim}{z}}{\partial q} + \frac{\partial \underset{\sim}{f}}{\partial q}. \tag{14}$$

This equation is valid if q is independent of t. Now, provided that the dependent variable $\underset{\sim}{z}$ is continuous and differentiable in both t and q, the order of the differentiation in Eq. (14) can be interchanged so that

$$\frac{\partial}{\partial t}\,\frac{\partial \underset{\sim}{z}}{\partial q} = \frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{z}}\,\frac{\partial \underset{\sim}{z}}{\partial q} + \frac{\partial \underset{\sim}{f}}{\partial q} \tag{15}$$

clearly, the derivative $\partial \underset{\sim}{z} / \partial q$ reflects the influence of changes in the parameter on the solution $\underset{\sim}{z}$ and is referred to the sensitivity function. Letting $\underset{\sim}{S} = \partial \underset{\sim}{z} / \partial q$, Eq. (15) becomes

$$\dot{\underset{\sim}{S}} = \frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{z}}\,\underset{\sim}{S} + \frac{\partial \underset{\sim}{f}}{\partial q}, \quad \underset{\sim}{S}(o) = \underset{\sim}{0} \tag{16}$$

Eqs. (16) are termed the "sensitivity equations" whose solutions are the sensitivity functions $\underset{\sim}{S} = \partial \underset{\sim}{z} / \partial q$.

Eqs. (13) and (16) are solved simultaneously to obtain the seismic response of the system and their time history sensitivity functions.

Computer Solution

A computer program was written to perform the calculations required to compute the natural damped frequencies, their sensitivities, the mode shapes, artificial earthquake records, seismic responses, and their time history sensitivity functions. The program was written in the FORTRAN IV and was run on UNIVAC Computer System.

3.0 RESULTS

The objective of this section is to report some of the analytical results on the dynamic interaction of structural-equipment system. The numerical analysis is done in two phases. In the first phase a free-vibration sensitivity analysis is carried out to examine the validity of the code decoupling criteria for the case of multi-degree of freedom system and to investigate the influence of the light-steel on the dynamic interaction of the structure-equipment system. In the second phase, the seismic response of the structural-equipment system is investigated to illustrate the dynamic interaction effects during a strong ground motion. In the following paragraphs, the results are summarized and discussed briefly. For convenience of reference, the parameters that are used in the following discussion are defined as follows:

1. M_r = mass ratio = M_e/M_f, where M_e is the equipment mass (or equipment mass + light-steel mass) and M_f is the floor mass.

2. F_r = normalized frequency ratio = W_{ei}/W_e (max), where W_{ei} is the equipment frequency. In this study the range of W_{ei} is given by 8 rad/sec $\leq$ W_{ei} $\geq$ 200 rad/sec, and W_e (max) = 200 rad/sec.

It should be noted that the basic assumption used herein is that decoupling is allowed if the variations in the response of the coupled system are less than 10 percent of that of the uncoupled system.

3.1 Free Vibration

The relation between M_r and F_r, based on the code formulae, is shown in Fig (2). The same relationship is plotted in Fig (3), based on the complex interpolation-sensitivity analysis, as discussed in Section 2, when the equipment is attached to different locations of the structural system. Further, by comparing these results, as illustrated in Fig (4), the following observations can be made:

1. Using the first mode (code) may lead to non-conservative results. However, a very conservative solution would result from using higher modes.

2. The lower bound solution for M_r occurs when the equipment is attached to the top floor.

The influence of the light-steel on the dynamic interaction is illustrated in Fig (5). The light-steel is assumed to be stiff and have a constant frequency of 220 rad/sec. Further, the practical limit of the light-steel mass is assumed, herein, to be about 20 percent of the equipment mass. By means of the curves in Fig (5) the following observations can be made:

1. The presence of light-steel tend to reduce the mass ratio M_r.

2. At $F_r = 0.137$ (first resonance), the mass of the equipment is about 20 percent less than that when the equipment is directly attached to the structural system.

3.2 Seismic Response

To illustrate the influence of the equipment-light-steel system on the dynamic interaction, the acceleration time history of the top floor of the structure, structure-equipment, and structure-light-steel-equipment systems, under the action of the strong ground acceleration of Fig (6), are plotted in Figs (7), (8) and (9), respectively. As the figures suggest, the equipment bears a strong effect on the response of the structure and that the light-steel has a negligible effect on the response.

4.0 CONCLUSIONS

Based on the studied system, Fig (1), the following preliminary conclusions may be reached:

1. The code formulae are generally conservative when the higher modes are used.

2. The light-steel tends to reduce the mass ratio. It bears, however, a small effect on the seismic response. This conclusion is based on a stiff light-steel system. The results could be different if a flexible light-steel system is used.

3. The sensitivity approach presented in this paper is a novel approach in application to structures subjected to seismic loads. For a safety related system, it is useful to know the parameter sensitivity prior to its implementation or to reduce the sensitivity systematically if this turns out to be necessary

(safety related system should be insensitive to parameter variations). Therefore, parameter sensitivity computations should be part of the design to any safety related structural or mechanical system.

5.0 REFERENCES

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5. Singhal, K., and Vlach, J., "Accuracy and Speed of Real and Complex Interpolation", Arch for Computing, Vol II, pp 147-158, 1973.

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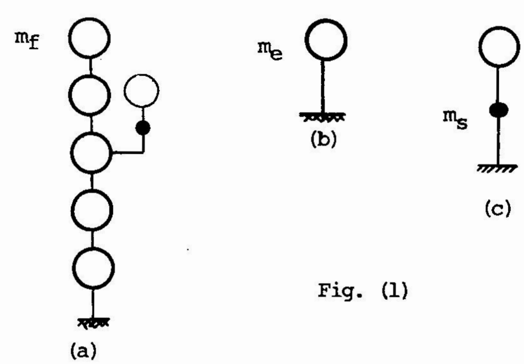

Fig. (1)

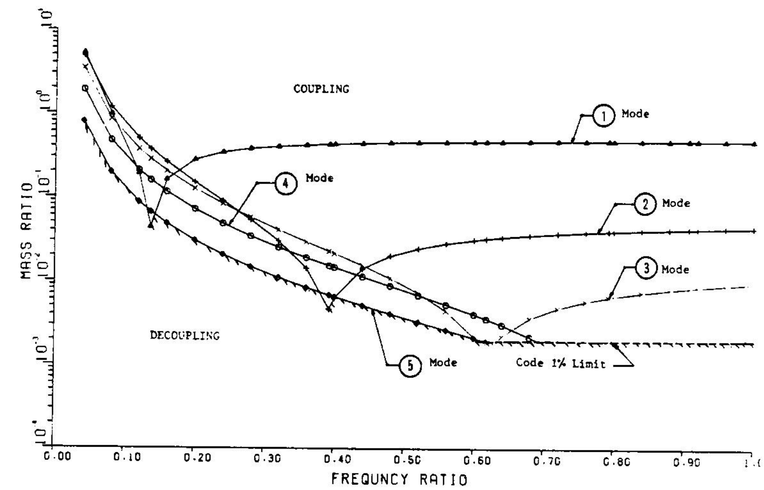
COUPLING
1 Mode
4 Mode
2 Mode
3 Mode
MASS RATIO
DECOUPLING
5 Mode
Code 1% Limit
FREQUNCY RATIO

Fig. (2)

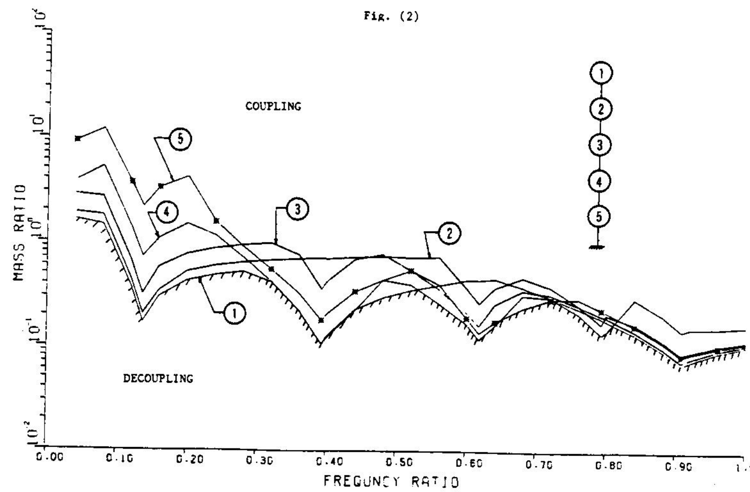
COUPLING
MASS RATIO
DECOUPLING
FREQUNCY RATIO

Fig. (3)

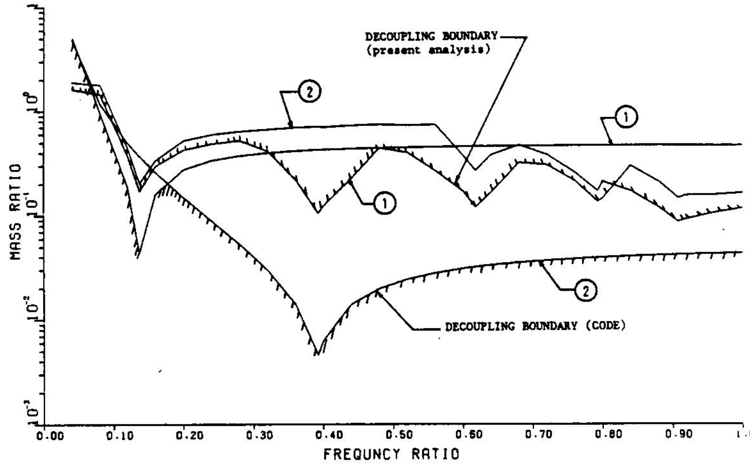
DECOUPLING BOUNDARY
(present analysis)
DECOUPLING BOUNDARY (CODE)
MASS RATIO
FREQUNCY RATIO

Fig. (4)

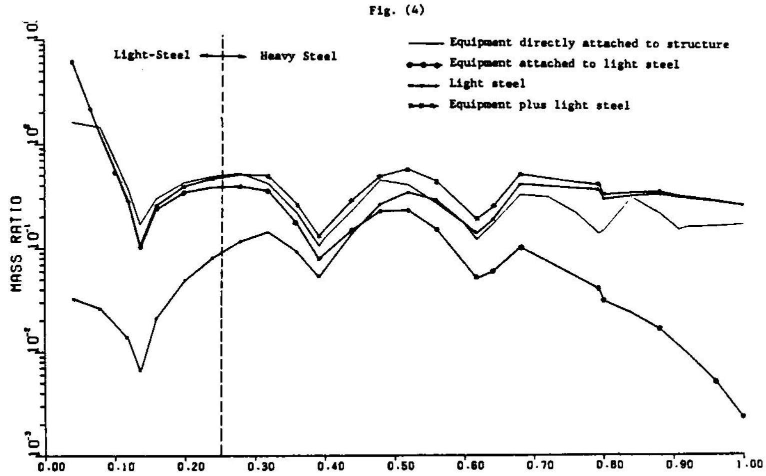
Light-Steel
Heavy Steel
Equipment directly attached to structure
Equipment attached to light steel
Light steel
Equipment plus light steel
MASS RATIO
FREQUNCY RATIO

Fig. (5)

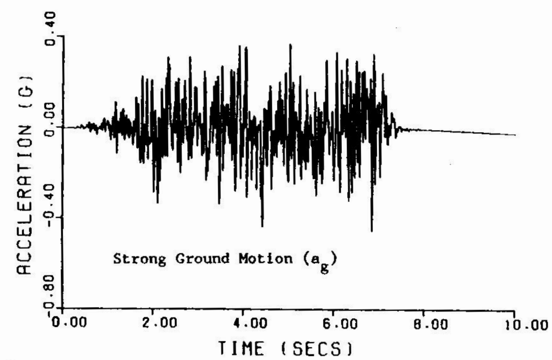

Fig. (6)

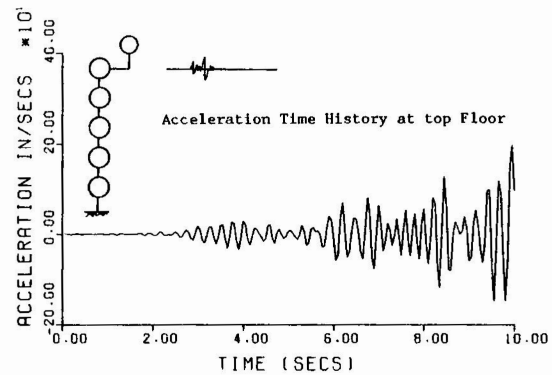

Fig. (8)

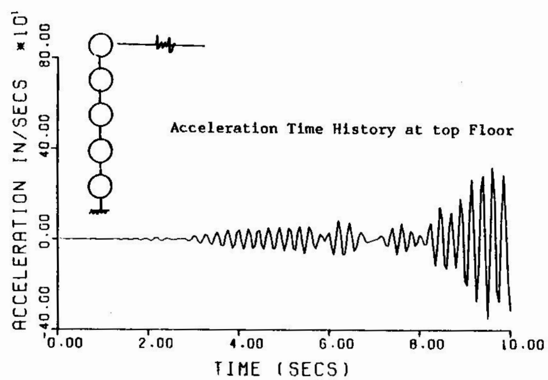

Fig. (7)

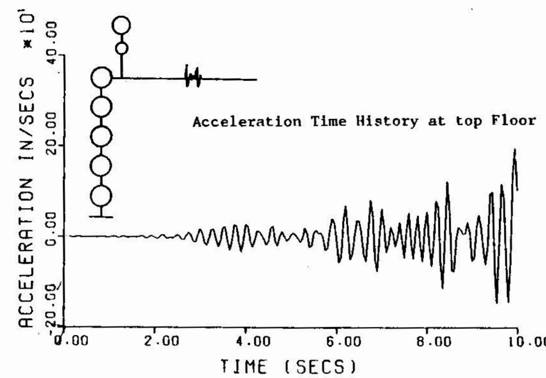

Fig. (9)